Homework Assignment #9

Non-invasive Imaging Techniques: Magnetic Resonance Imaging
(Hendee 23, 24)

Due December 4th

1. (5 points) Determine the fraction of spins aligned with an external magnetic field at body temperature (310 K), for a 2.0 T magnetic field for the case when
   a. The spins are electrons
   b. The spins are Phosphorous-31 nuclei

2. (5 points) The magnetic field at a distance of 0.15 nm from a proton is $4 \times 10^{-4}$ T. What change in Larmor frequency does this $\Delta B$ cause? How long will it take for a phase difference of $\pi$ radians to occur between a precessing spin feeling this extra field, and one that is not?

3. (5 points) Dephasing of the transverse magnetization during MRI can occur either because of spin-spin relaxation, specified by the so-called “non-recoverable” $T_2$ relaxation time, or because of imperfections in the magnetic field. If the magnetic field variation is $\Delta B_{\text{ext}}$ then the measured transverse relaxation time $T_2^*$, will depend on both of these factors:

   \[ \frac{1}{T_2^*} = \frac{1}{T_2} + \frac{\gamma \Delta B_{\text{ext}}}{2}. \]

   a. What is the effective relaxation time associated with the magnetic field variation for a proton in a magnetic field of 1.5 T with a uniformity of 1 part per million?
   b. The non-recoverable relaxation time of brain is about 2.5 ms. What dominates the measured transverse relaxation time in brain?

4. (10 points) This problem is a walk-through estimation of the energy absorption in a patient undergoing MRI. From this, you can get an idea of the "thermal dose" to the patient, which you can also measure in J/kg. You may want to refer back to a general physics textbook on electricity and magnetism for some of this.

   Treat the body as a uniform cylindrical conductor of Radius $R$ with electrical conductivity $\sigma$. Assume the magnetic field passing through the cylinder (through the end-caps, along the axis of the cylinder) is $B(t) = B_0 \cos(\omega_0 t)$.

   a. Calculate the magnetic flux passing through a circular surface of radius $r$ ($0 < r < R$)
   b. Using Faraday's law of induction, determine the electric field at $E(r)$.
   c. Use Ohm's law in the form $j = \sigma E$ ( $j$ is the electric current / area ) to show that the average power dissipated per unit volume of the material is:

      \[ p = \frac{\sigma E_0^2}{2} = \frac{\sigma R^2 \omega_0^2 B_0^2}{8}. \]

   d. If the radio-frequency signal is on for a period of time $\Delta t$ during a cycle period of $T_R$, revise your answer to part c to include the duty cycle of the pulse.
e. If the mass density of the material is $\rho$, calculate the specific absorption rate (SAR) which is defined as the power dissipated per unit mass.

f. For an RF pulse that causes a spin rotation through an angle of $\theta$ when the axial magnetic field strength is $B_0$, modify your answer in e to express the SAR in terms of $\Delta t$, $T_R$, $B_0$, $\theta$, $R$, $\rho$, and $\sigma$.

g. Use typical values for the human body $R=0.17$ m, $\sigma = 0.3$ S/m to evaluate this expression for a $\pi/2$ pulse.

h. For $B_0 = 1.5$ T and $\text{SAR} < 0.4$ W/kg, determine the minimum value of $\Delta t$ for $T_R = 1s$.

i. Find $B_1$ for the conditions in h.

j. For a 180 degree pulse, determine the thermal dose in J/kg.

5. (5 points) A certain MRI machine has a static magnetic field strength of 2 T. Spins are excited by applying a field gradient of 3 mT/m. If the slice is to be 5 mm thick, what is the Larmor frequency and the spread in frequencies required?